Intersection Graphs and Path Representations

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The study of intersection graphs — graphs arising as intersection patterns of structured families — has a long history, and a recurring theme is the question of optimal representations: given a graph G, how small can the underlying ground set be? For set representations, this is equivalent to asking for the minimum number of cliques needed to cover the edges of G, a question settled by Hall (1941) and independently by Erdős–Goodman–Pósa (1966).

In this talk, we study an analogous question for a different kind of representation. A *path representation* of G is a graph H together with an edge-partition of H into nontrivial paths such that $\{u, v\}$ is an edge of G if and only if the corresponding paths P_u and P_v share a vertex in H. We impose a natural non-degeneracy condition that no two paths in the partition of H can be concatenated, and define the *path representation number* $\mathsf{prn}(G)$ as the minimum number of vertices in any such "proper" path representation of G.

It turns out that every graph admits a proper path representation. We give bounds on $\mathsf{prn}(G)$ in terms of the order and size of G, as well as other well-known graph parameters such as the clique cover number, $\mathsf{cc}(G)$, the clique partition number, $\mathsf{cp}(G)$, and the clique number, $\omega(G)$. We investigate proper path representations of various classes of graphs, including complete bipartite graphs, complete graphs, the Erdős–Rényi random graph $G(n, p)$, and triangle-free graphs, particularly trees. We will also discuss a number of open problems that arise naturally from this work.

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